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Replicon modes and stability of critical behaviour of disordered systems with respect to the continuous replica symmetry breaking

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Abstract
A field-theory approach is used to investigate the 'spin-glass effects' on the critical behaviour of systems with weak temperature-like quenched disorder. The renormalization group (RG) analysis of the effective Hamiltonian of a model with replica symmetry breaking (RSB) potentials of a general type is carried out in the two-loop approximation. The fixed point (FP) stability, recently found within the one-step RSB RG treatment, is further explored in terms of replicon eigenvalues. We find that the traditional FPs, which are usually considered to describe the disorder-induced universal critical behaviour, remain stable when the continuous RSB modes are taken into account.

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1. Introduction

The spin-glass effects on the critical behaviour of weakly disordered systems have received intense attention during the last several years [1–7]. It is well known that the replica method is one of the very few general analytical methods which can be used to investigate disordered systems [8]. It enables one to average over disorder and employ standard field-theory techniques, such as the RG analysis. Most of the results obtained using the RG methods implicitly assume replica symmetry (RS). However, as is well known from the large body of works on glass-like systems, the Parisi RSB scheme is required to obtain the correct low temperature solutions of random models which are characterized by a macroscopic number of ground states [8, 9]. Recently, Le Doussal and Giamarchi [1] pointed out that although the RG treats fluctuations exactly, it has to be incorporated with RSB to escape a risk that it will miss the physics associated with the existence of a large number of metastable states in certain disordered systems.

Dotsenko *et al* [2–4] studied the RSB effects on the critical behaviour of disordered p-component magnets to one-loop order using ε expansion. They found that for $p < 4$ the

FPs, which are usually considered to describe the disorder-induced universal critical behaviour, are unstable with respect to the introduction of RSB potentials. The one-step RSB ansatz [8] allowed them to find the new FPs which turned out to be stable within the one-step RSB subspace, but the further analysis showed that for a general type of RSB there exists no stable FPs and the RG flows lead to the strong coupling regime [3]. However, the numerous investigations of pure and disordered systems performed in the two-loop and higher orders of the approximation prove that the predictions made in the lowest order of the approximation, especially on the basis of the ε expansion, can differ strongly from the real critical behaviour [10]. Therefore, the results of the RSB effect investigation in [2–4] must be revised with the use of more accurate approximations.

In [5] the one-step RSB effects on the critical behaviour have been reconsidered using the field-theory approach in the two-loop approximation. It was shown that the RS FPs are stable with respect to the introduction of the one-step RSB potentials. However, it is likely that the RSB in disordered systems should be of a general type rather than in the one-step RSB class [6]. Therefore, one cannot be sure that an FP scenario obtained by means of the one-step RSB ansatz describes adequately the spin-glass effects on criticality. In the present work, we re-investigate the RSB effects on the critical behaviour extending the field-theoretic description given in [5] to the case of the RSB structure of a general type.

2. Model and renormalization

The starting point of the field-theory approach to the study of spin systems near critical point in the presence of weak quenched disorder is the Ginzburg–Landau–Wilson Hamiltonian

$$H = \int \mathrm{d}^d r \left\{ \frac{1}{2} \sum_{i=1}^{p} [\nabla \phi_i(r)]^2 + \frac{1}{2} \big[\mu_0^2 - \delta\tau(r)\big] \sum_{i=1}^{p} \phi_i^2(r) + \frac{1}{4} v \sum_{i,j=1}^{p} \phi_i^2(r) \phi_j^2(r) \right\} \tag{1}$$

where $\phi_i(r)$ is the p-component order parameter. The quenched disorder is described by random fluctuations of the effective transition temperature $\delta\tau(r)$ whose probability distribution is taken to be symmetric and Gaussian,

$$P[\delta\tau] = A \exp\left[-\frac{1}{4u} \int \mathrm{d}^d r (\delta\tau(r))^2 \right] \tag{2}$$

where u is the small parameter which is proportional to the concentration of defects; and A is the normalization constant. To carry out the averaging over disorder we use the standard replica trick. After replicating, we get the effective Hamiltonian

$$H_n = \int \mathrm{d}^d r \left\{ \frac{1}{2} \sum_{i=1}^{p} \sum_{a=1}^{n} \big[\nabla \phi_i^a(r)\big]^2 + \frac{1}{2} \mu_0^2 \sum_{i=1}^{p} \sum_{a=1}^{n} \big[\phi_i^a(r)\big]^2 \right.$$
$$\left. + \frac{1}{4} \sum_{i,j=1}^{p} \sum_{a,b=1}^{n} v_{ab} \big[\phi_i^a(r)\big]^2 \big[\phi_j^b(r)\big]^2 \right\} \tag{3}$$

which is a functional of n replications of the original order parameter with an additional vertex u in the replica symmetric matrix $v_{ab} = v\delta_{ab} - u$. The properties of the original disordered system are obtained in the replica number limit $n \to 0$.

As pointed out in [2], near the critical point the system has a macroscopic number of the local minima solutions of the saddle-point equation corresponding to the Hamiltonian (1). The traditional RG treatment of the replicated Hamiltonian (3) cannot take into account the existence of these local minima so that the direct application of the RS RG scheme may be questioned. It was argued that spontaneous RSB can occur due to the interaction of the fluctuating fields with the local non-perturbative degrees of freedom coming from the multiple local minima solutions of the saddle-point equations. It was shown that the summation over these solutions in the replica partition function can provide the additional non-trivial RSB potential $\sum_{a,b} v_{ab}\phi_a^2\phi_b^2$ in which the matrix v_{ab} has the Parisi RSB structure [8].

We now realize the field-theoretic RG analysis of the effective Hamiltonian (3) with the RSB potential in the two-loop approximation. We use the massive field-theory scheme with renormalization of the one-particle irreducible (1PI) vertex functions $\Gamma_0^{(L,N)}\big(k_1,\ldots,k_L;q_1,\ldots,q_N;\mu_0^2,\{v_{ab}\}\big)$ at non-zero mass and zero external momenta [11]. The 1PI vertex functions are defined by

$$\delta\left(\sum k_i+\sum q_j\right)\Gamma_0^{(L,N)}\left(\{k\};\{q\};\mu_0^2,\{v_{ab}\}\right)=\int \mathrm{e}^{\mathrm{i}(k_i r_i+q_j\hat{r}_j)} \\ \times\langle\phi(r_1)\ldots\phi(r_N)\phi^2(\hat{r}_1)\ldots\phi^2(\hat{r}_L)\rangle_{1\mathrm{PI}}\,\mathrm{d}^d r_1\ldots\mathrm{d}^d r_N\,\mathrm{d}^d\hat{r}_1\ldots\mathrm{d}^d\hat{r}_L \tag{4}$$

where $\{q\},\{k\}$ are the sets of external momenta, the brackets $\langle\ldots\rangle$ denote the averaging performed with the effective Hamiltonian (3), and for the sake of simplicity the replica and vector indices have been omitted. The Feynman diagrams that contribute to the vertex functions (4) involve the momentum integration and depend on the microscopic cut-off Λ_0. To study the critical domain which corresponds to the large cut-off limit ($\Lambda_0\to\infty$) we have to renormalize the theory in order to absorb the divergences of diagrams in a change of parameters and to obtain meaningful expressions for the correlation functions. Introducing the renormalization factors for the fields Z_ϕ, Z_{ϕ^2}, the renormalized vertex functions $\Gamma_R^{(L,N)}$ are expressed in terms of the bare vertex functions as follows,

$$\Gamma_R^{(L,N)}(\{k\};\{q\};\mu^2,\{g_{ab}\})=Z_\phi^{N/2-L}Z_{\phi^2}^L\Gamma_0^{(L,N)}\left(\{k\};\{q\};\mu_0^2,\{v_{ab}\}\right) \tag{5}$$

where μ is the renormalized mass and g_{ab} is the renormalized matrix of couplings. To define the renormalization scheme completely, one has to impose the renormalization conditions for the renormalized vertex functions:

$$\begin{aligned}
&\Gamma_R^{(0,2)}(k,-k;\mu^2,\{g_{ab}\})\Big|_{k=0}=\mu^2\\
&\frac{\partial}{\partial k^2}\Gamma_R^{(0,2)}(k,-k;\mu^2,\{g_{ab}\})\Big|_{k=0}=1\\
&\Gamma_{abR}^{(0,4)}(k_1,k_2,k_3,k_4;\mu^2,\{g_{ab}\})\Big|_{k_i=0}=\mu^{4-d}g_{ab}\\
&\Gamma_R^{(1,2)}(k,-k;q;\mu^2,\{g_{ab}\})\Big|_{k=q=0}=1.
\end{aligned} \tag{6}$$

The scaling behaviour in the critical domain is described by the homogeneous Callan–Symanzik equation for the vertex functions [11],

$$\left[\mu\frac{\partial}{\partial\mu}+\sum_{a'b'}\beta_{a'b'}(\{g_{ab}\})\frac{\partial}{\partial g_{a'b'}}-\left(\frac{N}{2}-L\right)\gamma_\phi(\{g_{ab}\}) \\ -L\gamma_{\phi^2}(\{g_{ab}\})\right]\Gamma_R^{(L,N)}(\{k\};\{q\};\mu^2,\{g_{ab}\})=0 \tag{7}$$

with the coefficients given by

$$\beta_{ab}(\{g_{ab}\})=\frac{\partial g_{ab}}{\partial\ln\mu}\bigg|_{\{v_{ab}\},\mu_0}\qquad \gamma_\phi(\{g_{ab}\})=\frac{\partial Z_\phi}{\partial\ln\mu}\bigg|_{\{v_{ab}\},\mu_0} \\ \gamma_{\phi^2}(\{g_{ab}\})=\frac{\partial Z_{\phi^2}}{\partial\ln\mu}\bigg|_{\{v_{ab}\},\mu_0}. \tag{8}$$

Equations (6) imply that the renormalized matrix g_{ab} has the Parisi RSB structure, which is stipulated by the RSB structure of the bare matrix v_{ab}. According to the technique of the Parisi RSB algebra, in the limit $n \to 0$ the matrix g_{ab} is parameterized in terms of its diagonal elements $\tilde{g}$ and the off-diagonal function $g(x)$ defined in the interval $0 < x < 1$: $g_{ab} \to (\tilde{g}, g(x))$. The rules for operations with matrices g_{ab} are detailed in [4, 8]. In [5] the renormalization procedure (5)–(8) was carried out with the assumption that all matrices g_{ab} have the structure known as the one-step block-like RSB with the function $g(x)$ having the form

$$g(x) = \begin{cases} g_0 & \text{for} \quad 0 \leqslant x < x_0 \\ g_1 & \text{for} \quad x_0 < x \leqslant 1 \end{cases} \tag{9}$$

where the undetermined step parameter $x_0 \in [0, 1]$ is related to the initial disorder distribution. In the present work, we renormalize the theory to two-loop order assuming that the RSB structure is of the general Parisi type. The renormalization conditions (6) can be expressed in terms of $\tilde{g}$ and $g(x)$ as follows:

$$\begin{aligned}
&\Gamma_R^{(0,2)}[k, -k; \mu^2, \tilde{g}, g(x)]\Big|_{k=0} = \mu^2 \\
&\frac{\partial}{\partial k^2}\Gamma_R^{(0,2)}[k, -k; \mu^2, \tilde{g}, g(x)]\Big|_{k=0} = 1 \\
&\tilde{\Gamma}_R^{(0,4)}[k_1, k_2, k_3, k_4; \mu^2, \tilde{g}, g(x)]\Big|_{k_i=0} = \mu^{4-d}\tilde{g} \\
&\Gamma_R^{(0,4)}[k_1, k_2, k_3, k_4; \mu^2, \tilde{g}, g(x)]\Big|_{k_i=0} = \mu^{4-d}g(x) \\
&\Gamma_R^{(1,2)}[k, -k; q; \mu^2, \tilde{g}, g(x)]\Big|_{k=q=0} = 1.
\end{aligned} \tag{10}$$

Equations (10) are simultaneous integral equations, which show that we deal with the functional RG. To obtain the renormalized couplings and the renormalization factors we can solve equations (10) perturbatively order by order in $\tilde{g}$ and $g(x)$. The functional counterparts of equations (8) read

$$\tilde{\beta}[\tilde{g}, g(x)] = \frac{\partial \tilde{g}}{\partial \ln \mu}\Big|_{\tilde{v}, v(x), \mu_0} \qquad \beta[\tilde{g}, g(x)] = \frac{\partial g(x)}{\partial \ln \mu}\Big|_{\tilde{v}, v(x), \mu_0} \tag{11}$$

where $(\tilde{v}, v(x))$ represents the bare matrix v_{ab}. Equations (11) allow us to calculate the β functions as functionals of the renormalized parameters $\tilde{g}$ and $g(x)$. In the two-loop approximation we obtain the following expressions for the β functions,

$$\begin{aligned}
&\tilde{\beta}[\tilde{g}, g(x)] = \tilde{\gamma}_1 + \tilde{\gamma}_2 + \tilde{\gamma}_3 \qquad \tilde{\gamma}_1 = -\tilde{g} \qquad \tilde{\gamma}_2 = (8+p)\tilde{g}^2 - p\overline{g^2} \\
&\tilde{\gamma}_3 = ((8f - 40h + 20)p \\
&\qquad + 16f - 176h + 88)\tilde{g}^3 + 4p(6h - 2f - 3)\tilde{g}\overline{g^2} + 8p(1-2h)\overline{g^3}
\end{aligned} \tag{12a}$$

$$\begin{aligned}
&\beta[\tilde{g}, g(x)] = \gamma_1 + \gamma_2 + \gamma_3 \qquad \gamma_1 = -g(x) \\
&\gamma_2 = -4g^2(x) + (4+2p)\tilde{g}g(x) + 2pg(x) + pR_1(x) \\
&\gamma_3 = 16(1-2h)g^3(x) - (8p(1-4h) + 48 - 96h)\tilde{g}g^2(x) \\
&\qquad + ((8f - 48h + 28)p + 16f - 48h + 24)\tilde{g}^2 g(x) + 16p(1-2h)\tilde{g}\overline{g^1}g(x) \\
&\qquad + 8p(1-2h)\tilde{g}R_1(x) - 8p(1-4h)\overline{g^1}g^2(x) + 8phg(x)R_1(x) \\
&\qquad + 4p(4h - 2f - 3)\overline{g^2}g(x) - 8p(1-2h)R_2(x)
\end{aligned} \tag{12b}$$

where we have introduced $\overline{g^n} = \int_0^1 g^n(y)\,\mathrm{d}y$, $R_1(x) = \int_0^x [g(x) - g(y)]^2\,\mathrm{d}y$, $R_2(x) = \int_0^x [g(x)-g(y)][g^2(x)-g^2(y)]\,\mathrm{d}y$. The values of two-loop integrals are given by $f(d=3) = \frac{2}{27}$, $h(d=3) = \frac{2}{3}$, $f(d=2) = 0.114\,64$, $h(d=2) = 0.781\,29$. Analogous to [2–5], we changed $g_{ab} \to g_{ab}/J$ with $J = \int \mathrm{d}^d k/(k^2+1)^2$ and $g_{a\neq b} \to -g_{a\neq b}$ in equations (12). The RS situation corresponds to the case $g(x) = g_0$ in which equations (12) reduce immediately to equations obtained previously in [12] for vertices $v_1 = (p+8)(\tilde{g}+g_0)$ and $v_2 = 8g_0$.

It is well known that the expansions of RG functions in powers of coupling constants are asymptotic series. A self-consistent way to extract the required physical information from the obtained expressions (12) requires application of special resummation methods: Borel resummation accompanied by certain additional procedures (see [13] and references therein). We employ the Padé–Borel resummation method extended to the functional case. The resummation procedure consists of several steps: (i) starting from the functional $f[\tilde{g}, g(x)]$ in the form of a series in the auxiliary variable θ,

$$f[\tilde{g}, g(x)] = \sum_m \gamma_m \theta^m \to \sum_m \frac{\gamma_m}{m!}\theta^m \tag{13}$$

where γ_m denotes the contribution of order g^m, we construct its Borel transform (13); (ii) the Borel transform is extrapolated by a rational Padé approximant $[K/L](\theta)$ that is defined as the ratio of two polynomials both in the variable θ of degrees K and L such that the truncated Taylor expansion of the approximant is equal to that of the Borel transform of the functional f; (iii) the resummed function f_s is then calculated as the inverse Borel transform of this approximant:

$$f_s[\tilde{g}, g(x)] = \int_0^\infty \mathrm{d}\theta\, \mathrm{e}^{-\theta}[K/L](\theta). \tag{14}$$

In the two-loop approximation β functions (12) have the form $f = \gamma_1\theta + \gamma_2\theta^2 + \gamma_3\theta^3$. Using the [2/1] Padé approximant $[2/1](\theta) = \gamma_1\theta + 3\gamma_2^2\theta^2\big/(6\gamma_2 - 2\gamma_3\theta)$ we obtain

$$f_s[\tilde{g}, g(x)] = \gamma_1 - \frac{3\gamma_2^2}{2\gamma_3} - \frac{9\gamma_2^3}{2\gamma_3^2} + \frac{27\gamma_2^4}{2\gamma_3^3}\exp\left(-\frac{3\gamma_2}{\gamma_3}\right)\mathrm{Ei}\left(\frac{3\gamma_2}{\gamma_3}\right) \tag{15}$$

where $\mathrm{Ei}(x) = \mathcal{P.V.}\int_{-\infty}^x \mathrm{d}t\, \mathrm{e}^t/t$ is the exponential integral.

3. Stability of FPs and replicon eigenvalues

The nature of the critical behaviour is determined by the existence of a stable FP satisfying the following simultaneous equations

$$\tilde{\beta}_s[\tilde{g}^*, g^*(x)] = 0 \qquad \beta_s[\tilde{g}^*, g^*(x)] = 0. \tag{16}$$

Taking the derivative over x, we obtain from the last equation

$$\frac{\mathrm{d}}{\mathrm{d}x}\beta_s[\tilde{g}, g(x)] = \lambda[\tilde{g}, g(x)]g'(x). \tag{17}$$

Equation (17) implies that either the FP function $g^*(x)$ has the step-like structure, or it obeys the equation $\lambda[\tilde{g}^*, g^*(x)] = 0$. The simplest assumption is that the one-step RSB (9) occurs in the weakly disordered systems [2] so that the β functionals (12) reduce immediately to the three functions: $\beta_i(\tilde{g}, g_0, g_1)$, $i = 1, 2, 3$, obtained in [5]. The analysis of the β functions shows that there are three types of non-trivial FPs for different values of $p = 1, 2, 3$. Type I with $\tilde{g}^* \neq 0$, $g_0^* = g_1^* = 0$ corresponds to the RS FP of a pure system, type II with $\tilde{g}^* \neq 0$, $g_0^* = g_1^* \neq 0$ is a disorder-induced RS FP, and type III with $\tilde{g}^* \neq 0$, $g_0^* = 0$, $g_1^* \neq 0$

corresponds to the one-step RSB FP. The parameters $\tilde{g}^*$, g_0^*, g_1^*, describing the FPs position, depend on the coordinate of the step $x_0 \in [0, 1]$ and the obtained values of these parameters have been presented changing with the step $\Delta x_0 = 0.1$ in tables 1–4 of paper [5].

The type of critical behaviour of this disordered system for each value of p is determined by the stability of the corresponding FP. Within the one-step RSB parameter subspace, the requirement that the FP be stable reduces to the condition that the eigenvalues λ_i of the matrix

$$B_{i,j} = \frac{\partial \beta_i(\tilde{g}^*, g_0^*, g_1^*)}{\partial g_j} \tag{18}$$

lie in the right complex half-plane. The values λ_i calculated in [5] show that for three-dimensional (3D) and two-dimensional (2D) Ising models ($p = 1$) and 3D XY model ($p = 2$) the disorder-induced RS FPs are stable in the one-step RSB subspace, whereas the critical behaviour of 3D Heisenberg model ($p = 3$) is described by the RS FP of a pure system.

As shown in [3] the stability of the FP in the full RSB space may strongly differ from the stability in the one-step RSB subspace, so it is quite interesting to investigate the stability properties of the calculated early FPs with respect to the RSB modes of a general type. The β functions (12) obtained in the present work give the complete RG description of system (3) within the scope of the two-loop approximation. In order to investigate the full stability properties of FP we have to solve the RG flow equations

$$\begin{aligned} \mu \frac{\partial}{\partial \mu} \tilde{g}_\mu &= \tilde{\beta}_s[\tilde{g}_\mu, g_\mu(x)] \\ \mu \frac{\partial}{\partial \mu} g_\mu(x) &= \beta_s[\tilde{g}_\mu, g_\mu(x)] \end{aligned} \tag{19}$$

in the vicinity of the FP for $\mu \to 0$ [11]. This task seems to be very complicated in the two-loop approximation. Recently, an alternative method, the so-called 'replicon modes' approach, was suggested for this complex problem [6]. This approach allows for a very simple way to explore the stability properties with respect to the continuous RSB modes without solving equations (19) directly. Indeed, the usual way to examine the stability of the FP of a model with several coupling constants is to linearize and diagonalize the RG flow equations about this FP. The replicon modes approach is the generalization of this method to the specific system (3) with the infinite number of coupling constants. Following [6, 7] we introduce the eigenmode function

$$Q(x) = \frac{\mathrm{d}}{\mathrm{d}x}[g(x) - g^*(x)] \tag{20}$$

which characterizes the behaviour of the first derivative of $g(x)$ near the FP in the full RSB parameter space. In the case of a step-like RSB FP (RS FP, one-step or many-step RSB FP), as it follows from equation (17), the flow equation for the eigenmode function has the form

$$\mu \frac{\partial}{\partial \mu} Q_\mu(x) = \lambda_{\mathrm{rep}}(x) Q_\mu(x) \tag{21}$$

where we have introduced the replicon eigenvalues $\lambda_{\mathrm{rep}}(x)$ determining the scaling behaviour of the replicon modes $Q(x)$. Thus, the change from $g(x)$ to $Q(x)$ diagonalizes the flow equation about the step-like RSB FP. If $\lambda_{\mathrm{rep}}(x)$ is positive, $Q_\mu(x)$ decreases under the iteration of the RG transformation and the function $g(x)$ will approach the FP function $g^*(x)$. If $\lambda_{\mathrm{rep}}(x)$ is negative, $g(x)$ will deviate from its FP expression with a related instability along the continuous replicon mode directions in the parameter space. If $\lambda_{\mathrm{rep}}(x) = 0$, $g(x)$ will keep the initial shape and this should correspond to a marginal stability.

Table 1. Replicon eigenvalues $\lambda_{\rm rep}^{(1)}$.

		$d=3$			$d=2$
Type	x_0	$p=1$	$p=2$	$p=3$	$p=1$
I		−0.169 236	−0.001 673	0.131 538	−0.095 180
II		0.211 760	0.000 003	0.014 501	0.056 959
III	0.0	0.211 760	0.000 003	0.014 501	0.056 959
	0.1	0.245 366	0.000 092	0.009 909	0.066 102
	0.2	0.283 705	0.000 191	0.005 010	0.076 207
	0.3	0.326 606	0.000 302	−0.000 202	0.087 391
	0.4	0.375 427	0.000 425	−0.005 714	0.099 813
	0.5	0.430 949	0.000 566	−0.011 485	0.113 656
	0.6	0.494 819	0.000 728	−0.017 414	0.129 128
	0.7	0.568 823	0.000 913	−0.023 284	0.146 478
	0.8	0.654 381	0.001 130	−0.028 649	0.165 992
	0.9	0.751 314	0.001 388	−0.032 590	0.188 004
	1.0	0.857 323	0.001 695	−0.033 125	0.212 903

Using equations (19)–(21) we derive $\lambda_{\rm rep}(x) = \lambda[\tilde{g}^*, g^*(x)]$, where $\lambda[\tilde{g}, g(x)]$ is given by equation (17). Substituting into the latter the resummed β function (15) with the coefficients γ_i taken from equation (12*b*) we obtain the expression for the replicon eigenvalues function

$$\lambda_{\rm rep}(x) = \sum_{m=1}^{3} \left[\frac{1}{g'(x)}\frac{\partial \gamma_m}{\partial x}\right]_{\{\tilde{g}^*, g^*(x)\}} \frac{\partial f_s}{\partial \gamma_m}\Bigg|_{\{\gamma_i=\gamma_i[\tilde{g}^*, g^*(x)]\}}. \tag{22}$$

The set $\Lambda \equiv \{\lambda_1, \lambda_2, \lambda_3, \lambda_{\rm rep}(x)\}$ composed of the replicon eigenvalues and the eigenvalues of the stability matrix (18) yields a complete description of the FP stability properties in the full RSB parameter space [7]. We have calculated the replicon eigenvalues numerically for all FPs, which were found in the one-step RSB parameter subspace. Although the coefficient γ_3 in equation (12*b*) depends explicitly on x, it turns out that in the case of the one-step RSB FP (9) the replicon eigenvalues function (22) depends on x through $g^*(x)$ only, and therefore, it also has the one-step structure

$$\lambda_{\rm rep}(x) = \begin{cases} \lambda_{\rm rep}^{(0)} & \text{for} \quad 0 \leqslant x < x_0 \\ \lambda_{\rm rep}^{(1)} & \text{for} \quad x_0 < x \leqslant 1. \end{cases} \tag{23}$$

We have obtained that the three types of FPs are characterized by the replicon eigenvalues $\lambda_{\rm rep}^{(0)}$ coinciding with the corresponding eigenvalues λ_3 (see tables 1–4 in [5]) up to the numerical accuracy. The same identity occurs in the one-loop approximation [6, 7]. Nevertheless, there is no evidence that this identity holds in the higher orders of approximation. The obtained values of parameter $\lambda_{\rm rep}^{(1)}$ for the different types of FPs are presented in table 1. The analysis of Λ (see tables 1–4 of paper [5] and table 1 of this paper) calculated for the three types of FPs and for different values of $p = 1, 2, 3$ gives the following results.

(i) 2D and 3D Ising models ($d = 2, 3$; $p = 1$). Within the one-step RSB subspace the only stable FP is the disorder-induced RS FP (type II) with all $\mathrm{Re}\,\lambda_i > 0$, $i = 1, 2, 3$. In the case of the 3D Ising model ($d = 3$) the disorder-induced RS FP is characterized by the complex values λ_1 and λ_2 leading to RG flows which spiral around the FP [11] and giving rise to the oscillating corrections to scaling [14]. The type II FP remains stable when the continuous RSB modes are taken into account $\big(\lambda_{\rm rep}^{(0)}, \lambda_{\rm rep}^{(1)} > 0\big)$. The FP of a pure system (type I) and the disorder-induced RSB FP (type III) are unstable $\big(\lambda_3 \approx \lambda_{\rm rep}^{(0)} < 0\big)$. Therefore the critical

behaviour of weakly disordered 2D and 3D Ising systems is realized with the disorder-induced RS FP.

(ii) 3D XY model ($d = 3$; $p = 2$). Although the positive eigenvalues $\lambda_1, \lambda_2, \lambda_3, \lambda^{(0)}_{\text{rep}}, \lambda^{(1)}_{\text{rep}}$ calculated for the disorder-induced RS FP (type II) indicate that this FP is stable in the full RSB space, we have reasons to believe that in the higher orders of approximation the type I FP (which corresponds to the critical behaviour of a pure system) will become stable. Indeed, the small eigenvalues $\lambda_3, \lambda^{(1)}_{\text{rep}} \approx 3 \times 10^{-6}$ indicate the weak stability of the disorder-induced RS FP. On the other hand, the two-loop RG analysis of the disordered systems without RSB potentials [12] gives $p_c = 2.0114$ for the borderline between regions of stability for the disorder-induced FP ($p < p_c$) and the FP of a pure system ($p > p_c$), while the estimation of p_c performed in the six-loop approximation using the pseudo ε expansion gives $p_c = 1.912$ [15]. The latter result is in accordance with the Harris criterion, which suggests that due to the negative value of the specific heat exponent of the pure 3D XY model the critical behaviour of this model should be stable with respect to the influence of the quenched disorder at least in the absent of RSB effects.

(iii) 3D Heisenberg model ($d = 3$; $p = 3$). The disorder-induced RS FP is unstable due to the negative eigenvalues $\lambda_2, \lambda_3 = \lambda^{(0)}_{\text{rep}} < 0$. The disorder-induced RSB FP is stable in the directions corresponding to the continuous RSB modes for $0.1 \lesssim x_0 \lesssim 0.2$ $\left(\lambda^{(0)}_{\text{rep}}, \lambda^{(1)}_{\text{rep}} > 0\right)$, but this FP is unstable in the directions corresponding to the one-step RSB modes for any x_0 ($\lambda_2 < 0$). Additionally, both FPs are characterized by the non-physical values of coordinate $g_1^* < 0$, so that the RS FP of a pure system is the only stable FP in the full RSB space. Therefore the influence of weak quenched disorder on the critical behaviour of the 3D Heisenberg model is irrelevant even with taking into account the RSB effects.

4. Conclusions

The RG investigations carried out in the two-loop approximation show the stability of the critical behaviour of weakly disordered systems with respect to the introduction of potentials with the general Parisi RSB structure. In dilute Ising-like systems, the disorder-induced critical behaviour is realized with RS FP. The critical behaviour of XY- and Heisenberg-like systems is not affected by the weak quenched disorder.

Recently, the phase diagram of the random temperature Ising ferromagnet was studied within the framework of the Gaussian variational approximation [16]. The spin-glass phase separating the paramagnetic and ferromagnetic phases was found. It was shown that the transition from paramagnetic to spin-glass state is second order, whereas the transition between spin-glass and ferromagnetic states is first order. It was also shown that within the considered approximation there is no RSB in the spin-glass phase. If it is just the case, we argue that the paramagnetic to spin-glass transition is controlled by the traditional RS FP of a weakly disordered system and that slightly above this critical point the asymptotic behaviour is characterized by the critical exponents obtained previously for the diluted Ising model within the RS RG treatment. However, there are some strong arguments against the existence of an intermediate spin-glass phase [17] so that to define the nature of phase transition controlled by the obtained stable RS FP further investigation is clearly necessary.

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